Exercises with a *Mathematics* Dictionary

Students need help and encouragement in familiarising themselves with any work of reference if they are to make the most of it. The purpose of these pages is to provide a framework that will allow students to become familiar with a particular *Mathematics* Dictionary. For details of the dictionaries for which these exercises are intended, see the notes at the foot of the page.

Broadly speaking there are three stages in looking up any piece of information. These are: finding, understanding and using. The last two are separate but they tend to be fused together since the easiest way of checking that a new idea has been understood is by asking for it to be used. The familiarisation exercises suggested here are presented as four worksheets which can be photocopied. The worksheets are graduated.

Set A is mostly concerned with 'finding', and contains plenty of help. The page numbers needed are given for over half the questions, and the keyword which it is necessary to find in order to answer the question is printed in **bold**. No calculations are required.

Set B gives a little less help with 'finding', but every keyword is identified in **bold**. There is now more 'using' to be done. A basic calculator is adequate for the computation needed.

Set C does not give any help with page numbers and not all the keywords are identified in bold. A basic calculator will suffice for nearly all questions.

Set D At this level students are expected to identify nearly all the keywords for themselves or else try various possibilities. A scientific calculator is necessary for several questions.

The sheet most appropriate for an individual, group, or class will be for teachers to decide. Brief answers for most of the questions are given on page 6.

How this work may be used will depend on circumstances. If there is a class-set of dictionaries then the whole class could be engaged in the work at one time. Given that only a few dictionaries are available then it would be possible to work with one group at a time provided that the class organisation allowed this. Even if these exercises are not used in the classroom, individuals with their own copy of the dictionary might welcome the opportunity to do some structured work with it.

Students meeting the intended dictionary for the first time will undoubtedly want some help in finding their way via the Wordfinder, but this probably need be no more that a few oral exercises.

The dictionary might also be used as a starting-point for a piece of work. Some possibilities for this are given here on pages 7 and 8 as ***Matters arising*** . . .

These exercises were originally compiled for use with the *Oxford Mathematics Study Dictionary,* 1st edition 1996 - ISBN 019 914 551 2. This was subsequently published in the USA as *Barron's Mathematics Study Dictionary* 1998 - ISBN 07641 0303 2 - and nearly all of the work given here will fit with that edition also. These exercises **do not** fit the 1999 revised and enlarged edition for which a separate set of pages is available.

Help is given by printing in **bold** the word under which the information is to be found. In some cases the page number is also given. If it is not, then the word(s) will have to be looked up in the Wordfinder.

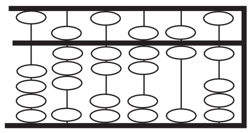
1. [page 12] Write down the size of any angle, in degrees, which is **obtuse**.

2. [page 14] Write down a group of five **consecutive numbers** starting with 11

3. [page 17] What is the value of a **discount** of 10% on £12 ?

4. [page 19] Which number is the **divisor** in the sum 798 ÷ 14 = 57 ?

5. [page 20] What is the number showing in the **soroban** drawn on the right?



6. [pages 30/31] Draw a **simple closed curve**.

7. [pages 32/33] How many cusps does a **cardioid** have ?

8. [page 35] Write down the next line of **Pascal's triangle** after the one given.

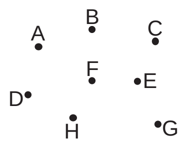
9. [page 36] Write down the **proper divisors** of 12

10. [page 74] What is the name of a **regular convex polyhedron** having 20 faces?

11. [page 40] Which number is the **numerator** in the fraction $\frac{7}{10}$?

12. [pages 34/35] Is it possible that **Mersenne** and **Fermat** could have known one another? Explain why.

13. [page 42] Write down the letters of the **3** dots which are **collinear** in the drawing on the right.



14. [page 65] What is the <u>total</u> value of these symbols written in the early **Roman number system**?

MCCLXXVIII

15. Give the **angle sum** of a quadrilateral.

16. How many faces does a **dodecahedron** have?

17. In the **bar chart** shown [on page 93], what was the shoe size found most often?

18. Which of these letters are **asymmetric**? A E F G J K L O Q U X Z

19. Use the temperature **conversion scale** to change 140°F into °C.

20. In an **isosceles triangle** how may edges are of the same length?

21. What **conversion factor** would you need to change gallons(UK) into litres?

22. Write down a number which is a **palindrome**.

23. Write out the value of a **trillion** in full.

24. Give another name for a **cube**.

25. How many lines of symmetry does the shape known as a **kite** have?

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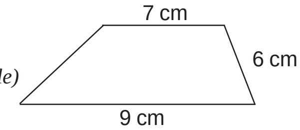
A calculator may be used. Where needed, use a value for π of 3.14.

1. [page 12] Write down the size of any angle, in degrees, which is **reflex**.
2. [page 15] Give the **digit sum** of 8025.
3. [page 22] Find the area of a **circle** which has a radius of 4 cm.
4. [page 28] A point is marked on a grid at (6,4). What is the value of the **abscissa**?
5. [page 32] What is the length of one arch of a **cycloid** when a = 3 cm?
6. [page 36] List the **proper factors** of 18.
7. [page 40] Write down an **improper fraction**.
8. [page 48] How many bytes are there in 6**Kb**?
9. [page 58] What is the **compass angle** of the direction described as East?
10. [page 61] What is the value of the **persistence** of the number 46 ?
11. What is the largest symbol used in the **hexadecimal** system?
12. Find the **angle sum** of a polygon having 5 edges.
13. Find the volume of a pyramid which has a **perpendicular height** of 12 cm and an area of base = 10 cm^2.
14. What is the total of the top row of the **magic square** given as an example of using primes only?
15. Use a **two-way table** for combining the throws of 2 dice, to find in how many ways a total of 6 can be made.
16. In the **pictogram** shown on page 93, how many pupils in Class 2 owned a computer?
17. Find the **arithmetic mean** of 7, 3, 2, 9, 11, and 10
18. In the **box and whisker diagram** what is the value of the median being shown in the example diagram?
19. Use a flow diagram to change a temperature of 92.3° on the **Fahrenheit scale** to a temperature on the Celsius scale.
20. What **conversion factor** would you use to change yards into metres? Use it to change 8 yards into metres.
21. Which **prefix** (for SI units) means "multiply by 1 000 000 000"?
22. List all the **prime numbers** between 80 and 100
23. What is the **complement** of an angle of 34° ?
24. Use a **formula** to find the area of the trapezium shown, which has parallel edges of length 9 and 7 cm, with a perpendicular height of 6 cm.

(not to scale)



25. One angle in a pair of **supplementary angles** is 27°. What is the size of the other angle?

Help is given in some cases by printing in **bold** the word(s) under which the required information being asked for is to be found.

A calculator may be used. Give all answers to a suitable degree of accuracy.

1. Write down the **transpose** of this matrix $\begin{pmatrix} {}^{-}3 & 5 \\ 7 & 1 \end{pmatrix}$
2. Which is bigger: a **googol** or a **centillion** ?
3. Find the area of a **sector** of a circle which has a radius of 7 cm and an angle at the centre of 50 degrees.
4. In the **histogram** given as an example, estimate the number of people in the 40 to 50 age group.
5. Give the value of -90°C as a temperature on the **Kelvin scale**.
6. What is the area enclosed by a **cardioid** when a = 3 cm?
7. The bearing of A from B is 217°. Give the **reciprocal bearing** of B from A.
8. Find the area of a regular **decagon** which has an edge length of 5 cm.
9. What is the **supplement** of an angle of 75 degrees?
10. Use **reverse percentages** to work out the original value of a camera which has had its price increased by 20% and is now marked at £102
11. Give the **digital root** of the number 16437
12. Work out the volume of a **frustum of a pyramid** when A = 20 cm^2 B = 45 cm^2 and the distance between the faces is 10cm.
13. Give the value of the 11th and 12th terms in the **Lucas sequence**.
14. Find the area of an ellipse having the values a = 8 cm and b = 10 cm.
15. Convert 40 gallons(UK) into its equivalent in litres.
16. Work out the value of the Mersenne prime for the case when n = 5
17. Find a solution to the asterithm given as an example and write out the sum in full.
18. How many lines of symmetry does a hypocycloid have when n = 4 ?
19. Show that 2196 is a Harshad number.
20. From the combination table for ${}^{n}C_{r}$ read off how many ways there are of choosing 6 objects from 11
21. Calculate the area of curved surface of a cylinder having a height of 12 cm and a radius of 4 cm.
22. Name two quadrilaterals which can NEVER be cyclic.
23. In the stem and leaf plot shown as an example, what is the frequency of the data having a stem value of 2 ?
24. Change the polar coordinates (7.6, 58º) into Cartesian coordinates.
25. How many counters are needed to make a polygon number in the shape of a hexagon having eight counters along each edge?

Help is given, in some cases, by printing in **bold** the word(s) under which the required information is to be found.
A calculator may be used. Give all answers to a suitable degree of accuracy.

1. Write down two angles which form a **conjugate** pair.
2. Show, by **casting out 9's**, that $258 \times 47 = 12156$ is wrong.
3. Make a sketch of the cross-section of a square **antiprism** when a cut is made parallel to one of the square end faces.
4. Work out the value of the **combination** $^{13}C_9$ when all the objects are different.
5. Rework the example given under **iteration** and find the value of x_5 to 6 decimal places.
6. An oblong **rep-tile** is to be made having one edge of length 8cm. What must be the length of its other edge?
7. Find a solution to the **alphametic** given and write down the values of all the letters. Hint: E is 5; N is 6
8. Work out the **instalments** to be paid at monthly intervals on a loan of £500 for 1 year at a rate of interest of 2.5% compounded monthly.
9. From all the approximations given for π on page 68, which is closest to the correct value?
10. Find the area of a segment of a circle which has a radius of 7 cm and an angle at the centre of 50 degrees.
11. In the Fibonacci sequence find the value of F_{20}
12. A triangle has an area of 60cm^2. What is the area of its median triangle?
13. Convert 40 gallons(UK) into its equivalent in gallons(US).
14. How many times greater is the Earth to Sun distance than the Earth to Moon distance?
15. Work out the value of the trace of this matrix $\begin{pmatrix} 2 & ^-4 & 1 \\ 0 & ^-9 & 5 \\ 8 & 3 & 6 \end{pmatrix}$
16. What is the area enclosed by an astroid when $a = 5.4$ cm?
17. Use Ramanujan's formula to find the perimeter of an ellipse when $a = 8$ cm and $b = 10$ cm.
18. Show that Goldbach's conjecture works for the number 24
19. Which is bigger - a googol or a centillion - and how many times bigger is it?
20. A regular octagon has an edge length of 4cm. What is its area?
21. A regular dodecahedron has a volume of 600 cm^3. What is the length of one edge?
22. Show that, for the cubic equation used as an example under 'trial and improvement', 0.89 is a better value for the root than the one given.
23. In a circle there are two intersecting chords, MN and PQ, which cross at X. MX is 10cm, NX is 7 cm and PX is 5cm. What is the length of PQ?
24. Sketch an Argand diagram which goes from 0 to 5 on both axes and on it mark the positions of the three points A, B and C which represent these numbers. Label each point with the appropriate letter.

    A is $3 + 4i$ B is $\sqrt{-1}$ C is $\sqrt{-17}$
25. A cylinder has height of 20 cm. A piece of string 30 cm long is wound <u>once</u> around the cylinder to form a helix. The string starts at the bottom of the cylinder and finishes at the top, at a point exactly above where it started. What is the diameter of the cylinder?

Answers

The page numbers [p. -] are given here if not given in the question.

Set A

1. *Any value between* 90 & 180°
2. 11, 12, 13, 14, 15
3. £1.20
4. 14
5. 82761
6. *Suitable drawing*
7. 1
8. 1 8 28 56 70 56 28 8 1
9. 1, 2, 3, 4, 6
10. Icosahedron
11. 7
12. Yes. Same country and overlap of 47 years. *(They actually did.)*
13. A F G
14. 1278 *[1000+200+50+20+5+3]*
15. [p. 72] 360°
16. [p. 74] 12
17. [p. 93] 4
18. [p. 103] F G J K L Q
19. [p. 107] 60°C
20. [p. 112] 2
21. [p. 117] 4.54609
22. [p. 60] *Suitable example excluding those given.*
23. [p. 63] 1 000 000 000 000
24. [p. 74] (regular) hexahedron
25. [p. 80] 1

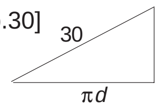
Set B

1. *Any value between* 180 & 360°
2. 15
3. 50.24 cm^2
4. Abscissa is 6
5. 24 cm
6. 1, 2, 3, 6, 9
7. *Example of improper fraction*
8. 6144
9. 090º
10. 46→24→8. Persistence is 2
11. [p. 64] F (≡ 15 *in base 10*)
12. [p. 72] 540º
13. [p. 78] 40 cm^3
14. [p. 83] 177
15. [p. 91] 5 ways
16. [p. 93] 20 pupils
17. [p. 94] 42 ÷ 6 = 7
18. [p. 96] 1.7 metres
19. [p. 106] 33.5ºC *[74.5 shows wrong use of calculator.]*
20. [p. 117] 0.9144; 7.3152 m
21. [p. 119] G *or* giga
22. [p. 37] 83, 89, 97
23. [p. 12] 56º
24. [p. 38] 48 cm^2
25. [p. 12] 153º

Set C

1. [p. 56] $\begin{pmatrix} ^-3 & 7 \\ 5 & 1 \end{pmatrix}$
2. [p. 63] centillion
3. [p. 22 or 38] 21.38 cm^2
4. [p. 93] about 75 000 people
5. [p. 106] 183.15 K
6. [p. 32] 169.6 cm^2
7. [p. 58] 37º
8. [p. 72] 192.4 cm^2
9. [p. 12] 105º
10. [p. 17] £85
11. [p. 15] 3
12. [p. 78 or 39] 316.7 cm^3
13. [p. 84] 184, 297
14. [p. 26 or 39] 251.3 cm^2
15. [p. 117] 181.8 litres
16. [p. 35] 31
17. [p. 83] 146 + 285 =431
18. [p. 33] 5
19. [p. 60] 2196 ÷ (2 + 1+ 9 + 6) =122
20. [p. 63] 462
21. [p. 24 *or* 39] 301.6 cm^2
22. [p. 80] rhombus and parallelogram
23. [p. 93] 5
24. [p. 28] (4.658, 6.005)
25. [p. 70] e = 6; n = 8;
    P_6 (8) = 120 ✓
    C_6 (8) = 169 ✗

Set D

1. [p.12] *2 values adding to 360*
2. [p.15] 6 × 2 *(d.r. = 3)* ≠ 6
3. [p.79] *An octagon*
4. [p.63] 715
5. [p.104] 1.584538
6. [p.89] Either 8 × $\sqrt{2}$ or 8 ÷ $\sqrt{2}$
    11.31 *or* 5.66 cm
7. [p.83] M = 1 O = 0 S = 9
    R = 8 D = 7 Y = 2
8. [p.17] £48.74
9. [p.68] $\sqrt{\sqrt{2143/22}}$ = 3.1415927
    but 355/113 is almost as close
10. [p.22 *or* 39] 2.612 cm^2
11. [p.84] 6765
12. [p.112] 60 ÷ 4 = 15 cm^2
13. [p.117] 40 × 1.2009
    = 48.036 galls(US)
14. [p.119] 0.389 × 10^3
    or 3.89 × 10^2
15. [p.56] -1
16. [p.32] 34.35 cm^2
17. [p.26] 56.72 cm^2
18. [p.34] 5 + 19 or 7 + 17
    or 11 + 13
19. [p.63] centillion; 10^{203}
20. [p.72] 77.25 cm^2
21. [p.74] 4.278 cm
22. [p.8] $^-$0.03 is closer to 0 than the 0.058 given
23. [p.44] QX = 14 ⇒ PQ = 19 cm
24. [p.67] *Sketch of Argand diagram*
25. [p.30]


    d = 7.118 cm

Many entries can be used as discussion points, set pieces of work, specific investigations or open-ended enquiries. On this page and the next are suggestions for generating ideas for further work.

Devise a **mnemonic** to remember the square root of 3; or to recall as many digits of π as you can; or to remember the order of the SI prefixes - perhaps one for those greater than 1 and another for those less than 1.

Construct a **direct proof** that adding an even and an odd number makes an odd number.
Construct a **visual proof** that adding an even and an odd number makes an odd number.
Do both of the preceding for subtraction or multiplication of, say, odd numbers and even numbers. Or odd and odd.

A **square** is defined as "a rectangle whose edges are all the same length". A direct result of that is *"It has four lines of symmetry"*. Would it be possible to define a square as "a quadrilateral having four lines of symmetry"? Would it be desirable or more useful? Find other definitions which can be turned around - and make sure they are true!

Investigate the **digital roots** of square numbers - or any other types of numbers.

An explanation is given of how to change **polar coordinates** into Cartesian coordinates. Explain how the reverse can be done.

Design and make a **conversion scale** for changing litres into gallons; or a *Shoppers Guide* for changing price per kilogram into price per pound.

Investigate the **locus** shown on page 31. The rectangle is 5 mm by 10 mm.

- What is the length of the curve drawn in red?
- What would be its length for a rectangle *x* mm by *y* mm?
- Study how the area being swept over by the string varies as it unwinds.

Almost every term explained under **recreational mathematics** could serve as the basis of either a single task or a major topic. For example:

- Make a set of **pentominoes** (cut out of card) and assemble them to make a complete rectangle.
- Find the 12 different **hexominoes** and assemble them into various shapes.
- Make a set of **Soma cubes** and put them together to make a cube.

The dictionary gives a lot of basic information, but much more can be obtained by building upon what is given. For instance:

- Make a formula or flow diagram to change temperatures on the Réamur scale (p.106) into their equivalent on the Celsius scale.
- How many barrels(oil) are there in one cubic metre?
- Use the **sine curve**, and a scientific calculator, to find the value of sine 2000° or some other value which is outside the range of your calculator.
- From the information given on p.48, deduce how many bytes there are in a Terabyte.
- Try to decide, and then describe, how the **suan pan** works for the simple case of adding two numbers. Then determine how the **soroban** manages the same task in spite of having two beads less on each wire. A simple model can be made by drawing suitable lines on a piece of paper and using counters to represent the beads.
- Design and make a **slide rule** (from two pieces of card) to multiply together any two whole numbers from 1 to 9. The calibration of these scales will have to be done by trial and error.

Compare the **decomposition** and **equal addition** methods of subtraction. Is one 'easier' than the other? Does the answer to that question depend upon what the numbers are? For instance, try both methods on 703 - 486.

Check the **nomogram** illustrated by doing some calculations and seeing how close the calculated values are to those given by the nomogram. Does it vary with the position you are using on the scale?

Investigate how you might find the size of an **ellipse** that was required to have a given area, or a given perimeter length, or both.

The diagram for **Eratosthenes' sieve** shows some lines have more prime numbers on them that others. Some have none at all. Look at other ways of setting the numbers out and try to find one that lines up the maximum number of primes.

Investigate the relationships between **polygon numbers**
(a) of the same type (b) of different types (c) of some other (invented) types.

Find out about the **odds** offered in horse-racing. For one particular race, change all the odds offered into probabilities and add them up. What do you notice? Does it always happen?

π With only a limited knowledge of trignometry of the right-angled triangle, Archimedes' method for approximating π (p.68) by 'squeezing' a circle between two polygons can be followed through for a hexagon, or some other polygon with a greater number of edges. The algorithm (p.69) can be worked through with a calculator for two passes, though the second pass will not be competely accurate. It is a valuable exercise in recording the different values as they change.

Most of the terms given under **number diversions** suggest starting points for further investigation. Some could be done by hand, and nearly all of them represent an obvious challenge for those capable of writing computer programs. There are many other opportunities for work on a computer.

On page 33 two epicycloids are shown, each having 5 cusps but, they have different n values. Why does this happen? Investigate the number of cusps an **epicycloid** (or a **hypocycloid**) will have when n is not a whole number. This is probably best done by writing a computer program for generating them. This can then be extended by attempting to draw the locus of a point which is NOT ON the circumference of the rolling circle but either inside or outside it. This creates the trochoids.

Most calculators will not give an exact value for the **factorial** of any number beyond that for 11. Produce a computer program that gives exact values up to 30!

Write a computer program to:

- produce a **proof by exhaustion** that "between every pair of square numbers less than X there is at least one prime number". Start with a small value of X and then see how large you can make it.
  How can you demonstate that your search is correct?
- produce **abundant** and **deficient numbers**. Which are there most of? Find an abundant number which is odd.
- draw a **spiral**.
- find the **persistence** of any number.
- evaluate the number of steps needed to produce **Kaprekar's constant** from any given four digits. Then investigate if there is a similar effect when three, or five, or six, digits are used
- evaluate **π** to (say) 1000 decimal places by use of Shank's method as outlined at the top of page 69.